



Printed Pages : 7

MCA-114

(Following Paper ID and Roll No. to be filled in your Answer Book)

**PAPER ID : 7304**

Roll No.

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**M.C.A.**

(Only for the candidates admitted/Readmitted in the session 2008-09)

**(SEM. I) EXAMINATION, 2008-09**

**DISCRETE MATHEMATICS**

*Time : 3 Hours]*

*[Total Marks : 100*

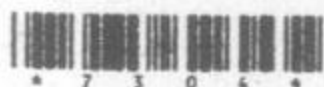
- 1 This question contains 10 objective type / fill 10×2=20  
in the blanks / true-false type questions.

Choose / fill / true or false correct answer.

- (a) If  $A$  and  $B$  are sets, then  $(A \cap B) \cup (A \cap \sim B)$

and  $A \cap (\sim A \cup B)$  are equal to :

- (i)  $A$  and  $B$
  - (ii)  $A$  and  $A \cap B$
  - (iii)  $A \cup B$  and  $A$
  - (iv)  $A \cup \sim B$  and  $A \cup B$
- (b) Let  $f: R \rightarrow R$  be given by  $f(x) = -x^2$  and  
 $g: R_+ \rightarrow R_+$  be given by  $g(x) = \sqrt{x}$  where  
 $R_+$  is the set of non negative real numbers and  
 $R$  is the set of real numbers. What are  $f \circ g$   
and  $g \circ f$  ?





- (i)  $x, -x$
- (ii)  $-x, x$
- (iii) not defined,  $-x$
- (iv)  $-x$ , not defined

(c) Let  $Q$  be the set of rational numbers and define

$$a * b = a + b - ab. \text{ Structure } \langle Q, * \rangle \text{ is}$$

- (i) Semigroup
  - (ii) Group
  - (iii) Monoid
  - (iv) None of these.
- (d) Which one of the following is false ?
- (i) The set of all bijective functions on a finite set forms a group under function composition.
  - (ii) The set  $\{1, 2, \dots, p-1\}$  forms a group under multiplication mod  $p$  where  $p$  is a prime number.
  - (iii) The set of all strings over a finite alphabet forms a group under concatenation.
  - (iv) A subset  $S \neq \Phi$  of  $G$  is a subgroup of the group  $\langle G, * \rangle$  if and only if for any pair of elements

$$a, b \in S, a * b^{-1} \in S.$$

(c) What is chromatic number of a graph ? Find the chromatic number for the complete bipartite graph  $K_{m, n}$ .

6 Attempt any one part of the following :

- (a) (i) Make a truth table for  $(p \wedge \sim p) \vee (\sim (q \wedge r))$ .
- (ii) Determine the truth value for each of the following statements. Assume  $x, y$  are elements of set of integers.

$$(1) \quad \forall x, y \quad x + y \text{ is even}$$

$$(2) \quad \exists x \forall y \quad x + y \text{ is even}$$

(b) Define a well formed formula. Show that the following equivalences :

$$(1) \quad P \rightarrow (Q \rightarrow R) \leftrightarrow P \rightarrow (\sim Q \vee R) \leftrightarrow (P \wedge Q) \rightarrow R$$

$$(2) \quad P \rightarrow (Q \vee R) \leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$$

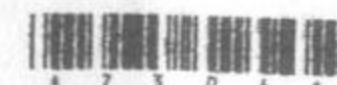
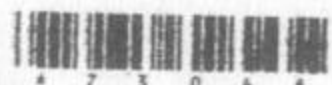
7 Attempt any one part of the following :

(a) Solve the recurrence relation

$$y_{n+2} - y_{n+1} - y_n = n^2$$

(b) Explain the following terms :

- (i) Pigeon hole principle
- (ii) Polya's counting theorem.





## SECTION - B

2 Attempt any three parts of the following :  $10 \times 3 = 30$

(a) (i) When is  $A - B = B - A$  ? - Explain.

(ii) Show that any positive integer  $n$  greater than or equal to 2 is either a prime or a products of primes.

(b) Let a binary operation  $*$  on  $G$  be defined by  $(a, b) * (c, d) = (ac, bc + d)$  for all ordered pairs  $(a, b)$  of real numbers,  $a \neq 0$ . Show that  $(G, *)$  is a non abelian group. Does that subset  $H$  of all those elements of  $G$  which are of the form  $(1, b)$  form a subgroup of  $G$  ?

(c) Let  $A$  be the set of factors of a particular positive integer  $m$  and let  $\leq$  be the relation divides i.e.

$$\leq = \{(x, y) \mid x \in A \text{ and } y \in A \text{ and } (x \text{ divides } y)\}$$

Draw the Hasse diagram for

(i)  $m = 30$

(ii)  $m = 12$

(iii)  $m = 45$ .

(d) (i) Show that :

$$((P \vee Q) \wedge \sim (\sim P \wedge (\sim Q \vee \sim R))) \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$$

is a tautology.



(ii) Let  $q(x, y, z)$  be the statement

" $x + y = z$ ". What are the truth values of the statements ?

$$\forall x \forall y \exists z q(x, y, z) \text{ and}$$

$$\exists z \forall x \forall y q(x, y, z) ?$$

(e) The generating function of a sequence

$a_0, a_1, a_2, \dots$  is the expression

$$A(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n + \dots \text{ using}$$

generating functions, solve the recurrence relation

$$a_n + 3a_{n-1} - 10a_{n-2} = 0 \text{ for } n \geq 2 \text{ and}$$

$$a_0 = 1, a_1 = 4.$$

## SECTION - C

$10 \times 5 = 50$

3 Attempt any **one** part of the following :

(a) Prove that the relation "congruence modulo  $m$ " given by

$$\equiv = \{(x, y) \mid x - y \text{ is divisible by } m\}$$

Over the set of positive integers is an equivalence relation. Also show that if  $x_1 \equiv y_1$  and  $x_2 \equiv y_2$  then

$$(x_1 + x_2) \equiv (y_1 + y_2)$$





- (b) (i) What do you mean by recursively defined functions? Give an example.
- (ii) How does an indirect proof technique differ from direct proof technique? - Explain.

4 Attempt any two parts of the following :

- (a) What do you mean by isomorphism of semigroups? How does an isomorphism of semigroups differ from an isomorphism of posets?
- (b) Let  $S = \{1, 3, 7, 9\}$  and  $G = (S, \text{multiplication mod } 10)$ . Determine all left and right cosets of the subgroup  $\{1, 9\}$ .
- (c) How does a field differ from a ring? Explain with example.

5 Attempt any two parts of the following :

- (a) Let  $L$  be a lattice. Prove that for every  $a, b$  and  $c$  in  $L$ , if  $a \leq b$  and  $c \leq d$ , then  $a \vee c \leq b \vee d$  and  $a \wedge c \leq b \wedge d$ .
- (b) Let  $(D_{63}, \leq)$  be lattice of all positive divisors of 63 and  $x \leq y$  means  $x/y$ . Prove or disprove the statement :  $(D_{63}, \leq)$  is a Boolean algebra.

- (e) Let  $X = \{2, 3, 6, 12, 24\}$ , let  $\leq$  be the partial order defined by  $x \leq y$  if  $x$  divides  $y$ . Number of edges in the Hasse diagram of  $(X, \leq)$  is

- (i) 3  
(ii) 4  
(iii) 9  
(iv) none of these

- (f) The proposition  $P \wedge (\sim P \vee Q)$  is

- (i) A tautology  
(ii) Logically equivalent to  $P \wedge Q$   
(iii) Logically equivalent of  $P \vee Q$   
(iv) A contradiction.

- (g) Maximum number of edges in a planar graph with  $n$  vertices is \_\_\_\_\_.

- (h) Let  $Q(x) : x + 1 < 4$ . Then  $\forall x Q(x)$  is \_\_\_\_\_, and  $\exists x Q(x)$  is \_\_\_\_\_.

- (i) The largest possible number of leaves in an  $n$ -tree of height  $k$  is \_\_\_\_\_.

- (j) If  $n$  pigeons are assigned to  $m$  pigeonholes then, one of the pigeonhole must contain at least \_\_\_\_\_ pigeons.

