(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 7304 Roll No.

M.C.A.

(Only for the candidates admitted/Readmitted in the session 2008-09)

(SEM. I) EXAMINATION, 2008-09

DISCRETE MATHEMATICS

Time : 3 Hours] *[Total Marks : 100*

1 This question contains 10 objective type / fill in the blanks / true-false type questions. Choose / fill / true or false correct answer. **10×2=20**

(a) If A and B are sets, then $(A \cap B) \cup (A \cap \sim B)$ and $A \cap (\sim A \cup B)$ are equal to :

(i) A and B

(ii) A and $A \cap B$

(iii) $A \cup B$ and A

(iv) $A \cup \sim B$ and $A \cup B$

(b) Let $f: R \to R$ be given by $f(x) = -x^2$ and $g: R_+ \to R_+$ be given by $g(x) = \sqrt{x}$ where R_+ is the set of non negative real numbers and R is the set of real numbers. What are fog and gof ?

(i) $x, -x$

(ii) $-x, x$

(iii) not defined, $-x$

(iv) $-x$, not defined

(c) Let Q be the set of rational numbers and define

$a * b = a + b - ab$. Structure $< Q, * >$ is

(i) Semigroup

(ii) Group

(iii) Monoid

(iv) None of these.

(d) Which one of the following is false ?

(i) The set of all bijective functions on a finite set forms a group under function composition.

(ii) The set $\{1, 2, \ldots\ldots, p-1\}$ forms a group under multiplication $\textbf{mod } p$ where p is a prime number.

(iii) The set of all strings over a finite alphabet forms a group under concatenation.

(iv) A subset $S \neq \Phi$ of G is a subgroup of the group $< G, * >$ if and only if for any pair of elements

$a, b \in S, \ a * b^{-1} \in S$.

(c) What is chromatic number of a graph ? Find the chromatic number for the complete bipartite graph $K_{m,\, n}$.

6 Attempt any one part of the following :

(a) (i) Make a truth table for $(p \wedge \sim p) \vee (\sim (q \wedge r))$.

(ii) Determine the truth value for each of the following statements. Assume x, y are elements of set of integers.

(1) $\forall x, y \quad x + y \text{ is even}$

(2) $\exists x \forall y \quad x + y \text{ is even}$

(b) Define a well formed formula. Show that the following equivalences :

(1) $P \rightarrow (Q \rightarrow R) \leftrightarrow P \rightarrow (\sim Q \vee R) \leftrightarrow (P \wedge Q) \rightarrow R$

(2) $P \rightarrow (Q \vee R) \leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$

7 Attempt any one part of the following :

(a) Solve the recurrence relation

$$y_{n+2} - y_{n+1} - y_n = n^2$$

(b) Explain the following terms :

(i) Pigeon hole principle

(ii) Polya's counting theorem.

SECTION - B

2 Attempt any three parts of the following : 10×3=30

(a) (i) When is $A - B = B - A$? - Explain.

(ii) Show that any positive integer n greater than or equal to 2 is either a prime or a products of primes.

(b) Let a binary operation $*$ on G be defined by $(a, b) * (c, d) = (ac, bc + d)$ for all ordered pairs (a, b) of real numbers, $a \neq 0$. Show that $(G, *)$ is a non abelian group. Does that subset H of all those elements of G which are of the form $(1, b)$ form a subgroup of G ?

(c) Let A be the set of factors of a particular positive integer m and let $\leq$ be the relation divides i.e.

$$\leq = \{(x, y) \mid x \in A \text{ and } y \in A \text{ and } (x \text{ divides } y)\}$$

Draw the Hasse diagram for

(i) $m = 30$

(ii) $m = 12$

(iii) $m = 45$.

(d) (i) Show that :

$$((P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R))) \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$$

is a tautology.

(ii) Let $q(x, y, z)$ be the statement "$x + y = z$". What are the truth values of the statements ?

$\forall x \forall y \ \exists z \ q(x, y, z)$ and

$\exists z \ \forall x \ \forall y \ q(x, y, z)$?

(e) The generating function of a sequence $a_0, a_1, a_2, \ldots$ is the expression

$A(x) = a_0 + a_1 x + a_2 x^2 \ldots + a_n x^n + \ldots$ using generating functions, solve the recurrence relation $a_n + 3a_{n-1} - 10a_{n-2} = 0$ for $n \geq 2$ and $a_0 = 1$, $a_1 = 4$.

SECTION - C

10×5=50

3 Attempt any **one** part of the following :

(a) Prove that the relation "congruence modulo m" given by

$$\equiv = \{(x, y) \mid x - y \text{ is divisible by } m\}$$

Over the set of positive integers is an equivalence relation. Also show that if $x_1 \equiv y_1$ and $x_2 \equiv y_2$ then

$(x_1 + x_2) = (y_1 + y_2)$

(b) (i) What do you mean by recursively defined functions ? Give an example.

(ii) How does a indirect proof technique differ from direct proof technique ? - Explain.

4 Attempt any two parts of the following :

(a) What do you mean by isomorphism of semigroups ? How does an isomorphism of semigroups differ from an isomorphism of posets ?

(b) Let $S = \{1, 3, 7, 9\}$ and $G = (S, \textit{multiplication} \ \textbf{mod 10})$. Determine all left and right cosets of the subgroup $\{1, 9\}$.

(c) How does a field differ from a ring ? Explain with example.

5 Attempt any **two** parts of the following :

(a) Let L be a lattice. Prove that for every a, b and c in L, if $a \le b$ and $c \le d$, then $a \vee c \le b \vee d$ and $a \wedge c \le b \wedge d$.

(b) Let $(D_{63}, \le)$ be lattice of all positive divisors of 63 and $x \le y$ means x / y. Prove or disprove the statement : $(D_{63}, \le)$ is a Boolean algebra.

(e) Let $X = \{2, 3, 6, 12, 24\}$, let $\le$ be the partial order defined by $x \le y$ is x divides y. Number of edges in the Hasse diagram of $(X, \le)$ is

(i) 3

(ii) 4

(iii) 9

(iv) none of these

(f) The proposition $P \wedge (\sim P \vee Q)$ is

(i) A tautology

(ii) Logically equivalent to $P \wedge Q$

(iii) Logically equivalent of $P \vee Q$

(iv) A contradiction.

(g) Maximum number of edges in a planar graph with n vertices is ________.

(h) Let $Q(x): x + 1 < 4$. Then $\forall x Q(x)$ is ________, and $\exists x\, Q(x)$ is ________.

(i) The largest possible number of leaves in an n-tree of height k is ________.

(j) If n pigeons are assigned to m pigeonholes then, one of the pigeonhole must contain at least ________ pigeons.